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ABSTRACT

Statistical invariance procedures provide a way of looking at the generalizability of research results from sample to sample when the research has not been validated by replication. This paper discusses the Procrustean Rotation invariance procedure following a canonical correlation analysis. The computer program RELATE is used to gauge the replicability and generalizability of the illustrative substantive research results. A Procrustean rotation forces orthogonal (uncorrelated) functions of factors to a best fit position after setting the factor vectors to unit length (1.0) in order to equalize the contribution of each factor vector to the determination of the amount of rotation necessary. This rotation technique can be used as a cross-validation procedure, splitting data from a single sample and comparing factor vectors for each half. The sample used to illustrate the procedure is from a study of leadership styles conducted by M. L. Tucker (1990) using data from 106 college faculty and administrators, with the university research sample split into two uneven data sets (n=48 and n=58). Two tables of illustrative data and an 11-item list of references are included. (SLD)

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APPLYING PROCRUSTEAN ROTATION TO EVALUATE THE GENERALIZABILITY OF CANONICAL ANALYSIS RESULTS

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ABSTRACT

Thoughtful researchers place importance on replicability of research results. Statistical invariance procedures provide a way of looking at the generalizability of research results from sample to sample when the research has not been validated by replication. The present paper provides a discussion of the Procrustean Rotation invariance procedure following a canonical correlations analysis. The computer program RELATE is utilized to gauge the replicability and generalizability of the illustrative substantive research results.

Researchers today still rely heavily on statistical significance testing to validate research findings. Yet, Tukey (1969) reminds researchers that it was never the intention of the statistical significance analysis to be substituted for replication of research.

> The modern test of significance, before which so many editors of psychological journals are reported to bow down, owes more to R. A. Fisher than to any other man. Yet Sir Ronald's standard of firm knowledge was not one very extremely significant result but rather the ability to repeatedly get results significant at 5%. Repetition is the basis for judging variability and significance and confidence. Repetition of results, each significant, is the basis, according to Fisher, of scientific truth. (p. 85)

Yet all research cannot be replicative ones. There is a need, then, to address invariance of initial research findings. Kerlinger (1986) reminds researchers that all research using sampling techniques is inherently subjected to sampling error. Even though survey information is generally found to be relatively accurate, "there is always the one chance in twenty or a hundred that an error...caused by minor fluctuations of chance may occur (p. 387). It is pertinent, then, that sample invariance issues be addressed. Thompson describes invariance procedures as looking at how generalizable results are from sample to sample, i.e., how much they <u>do not</u> vary from sample to sample. The logic of invariance analysis was summarized by Fish (1986), as an attempt to determine how stable the statistical results are likely to be across different samples.

> In the typical invariance procedure an analysis is performed separately on each of two subgroups into which the study

> sample has been divided, and the results are compared. When the results of an analysis are not comparable--i.e., not invariant--serious doubts about the generalizability of the results are in order. (pp. 65-66)

Invariance analysis provides more confidence that research results are stable and replicable across samples. The current study applied an invariance technique following a canonical correlation analysis. For readers unfamiliar with canonical correlation analysis, Thompson (1984) provides a very understandable explanation.

Like all correlational methods, canonical correlations analysis tends to capitalize on chance; in fact, Thompson (1981) asserts that most extraction methods achieve their criterion by such capitalization. For a factor analysis, Gorsuch (1983) recommends that, by randomly splitting a study sample in half and comparing the factors extracted from both data sets, an estimate of the magnitude of this capitalization can be found.

Procrustean Rotation

A Procrustean rotation can be used with any multivariate technique. The name "Procrustean" originates in Greek mythology. Procrustes, a son of Poseidon, forced travelers spending the night at his home to fit his bed by either cutting off their legs or stretching their bodies. Similarly, a Procrustean rotation forces orthogonal (uncorrelated) functions of factors to a "best fit" position after setting the factor vectors to unit length (1.0) "in order to equalize the contribution of each [factor vector] to the determination of the amount of rotation necessary" (Veldman, 1967, p. 238). This rotation technique can be used as a cross-validation procedure, splitting the data from a single sample and comparing the factor vectors from each half.

The use of factor analysis as a means of validity evaluation is well known. Thompson and Pitts (1981/1982) describe this cosine application as a rotation of calculated factors to a position of "best fit" with a target matrix that has been theoretically derived. The target matrix determines the expected number of factors and the expected correlation between each item and each factor. Thus, the cosines of the angles between the actual and the hypothetical measures can be interpreted as validity coefficients.

In this procedure, the original sample is split into two subsets, structure or function coefficients are generated, resulting in two sets of coefficients for comparison using the "best fit" rotation method. Thompson (1986) provides a detailed review of this empirical method developed by Kaiser, Hunka, and Bianchini (1969) for "relating" factors derived from different samples of data. This method consists of projecting the two sets of factors into the same factor space and calculating the cosines of the angles among the factors across the two solutions. These cosines provide a measure of the relatedness of the two sets of factors, and are similar to correlation coefficients.

Thompson (1981, 1986) affirms that these Procrustean rotation derived coefficients are analogous to test-retest coefficients and has called them invariance coefficients. They may also be utilized as adequacy coefficients for substantive interpretations.

Application of the Procrustean Rotation

The sample used for illustration of the Procrustean rotation is taken from a substantive study by Tucker (1990) of transformational, transactional, and laissez-faire leadership styles as predictors of follower satisfaction with leader, of follower perceptions of leader effectiveness, and of extra effort expended by the follower for a leader. Data were

gathered from 106 faculty and administrative respondents who rated their individual leaders located at a Southern, urban university (XYZ).

The Procrustean rotation of the standardized canonical function coefficients was utilized as an invariance procedure to further analyze the data at hand for generalizability. In particular, this research investigated the similarity of function equations for the predictor variable set across subsamples of XYZ research data. Although either function coefficients or structure coefficients may be submitted to a Procrustean rotation, for this particular application, function coefficients are used because the primary concern was to investigate the invariance of the predictor variable set, focusing on the invariant predictability of function equations used to produce variate scores.

This university research sample (N = 106) was split into two uneven data sets (n = 48, n = 58). Data from each subsample were entered into a separate canonical correlation analysis. The resulting predictor variable function coefficient matrices for Functions I, II, and III were submitted to a Procrustean rotation using RELATE (Veldman, 1967), as illustrated in Table 1. The standardized function coefficients for sample one (n = 48) were input as matrix A, and those from sample two as matrix B. The decision regarding which matrix is to be designated the target for "best fit" rotation is arbitrary. The cosines, or correlation coefficients, that were derived when projecting the two sets of data into the same factor space, provide the relatedness of the two separate subsamples. Thompson (1981, 1984, 1986) affirms that these coefficients are analogous to invariance coefficients and states that they may also be used for substantive interpretations as adequacy coefficients.

INSERT TABLE 1 ABOUT HERE

Although the main focus of interest is the resulting matrix cosines, test r's should be first consulted. These evaluate the relation of the given variables from the two data sets within the factor space, and must be suitable for the functions being rotated to "best fit" to be suitable. Reliability of the correlation of each canonical function, based on the two matrices, was provided in test r's by RELATE. The test variable r's, as shown in Table 2 for the three rotated matrices, produced by RELATE were .997, .998, and .998. These results indicate (since they are large) that the three canonical functions can be rotated in this manner.

With respect to the resulting cosines among the functions, generally, to be considered replicated, functions should have a cosine of roughly .8 or higher (Thompson & Pitts, 1981/1982). Kaiser et al (1969) suggests .85 is reasonable and Gorsuch (1983) recommends results greater than .93 as exceptional; however, Thompson (1986) presents empirically derived cutoffs as an alternative to the theoretically derived cutoffs formulated by others.

The cosines among each of the three functions for this canonical correlation analysis data are presented in Table 2. The cosines resulting from the Procrustean-rotated canonical standardized functions were all close to one, above the .93 exceptional range set by Gorsuch, thus confirming that the original results are sample "invariate." These Procrustean rotation invariance analysis results affirmed the invariance of this set of data for all three predictor variables on all three canonical functions.

INSERT TABLE 2 ABOUT HERE

This RELATE procedure produced an investigation of the similarity of function equations used to produce variate scores. The use of structure coefficients would have investigated "similarity of the distribution of variable variances across equations" (Thompson, 1984, p. 45).

However, Thompson (1984) reminds us that there is much yet to learn about the characteristics of various invariance estimates and suggests that researchers employ several procedures in obtaining an "upper bound" and "lower bound" of the degree of capitalization on sampling specificity. Thompson (p. 46) cautions of the importance to recognize that all procedures are "liberal" estimates of invariance when one data set is split into subgroups, because the...subgroups came from the same sample and the subgroups and their parameter estimates are therefore interdependent. A more conservative approach to the estimation of result invariance is provided when the canonical analysis involves two independent samples of subjects.

Nevertheless, it is imperative to derive some empirical evaluation of the generalizability of results, since the cumulation of findings across studies is the business of science.

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Table 1

Matrices Entered Into Procrustean Rotation:

Standardized Function Coefficients of Each Split-Sample Canonical Run

MATRIX A (TARGET MATRIX, n = 48):

Function I	Function II	Function III
0.98000	1.04100	0.64000
0.00700	1.34600	0.01500
0.02900	0.40000	1.17300

MATRIX B (n = 58):

Function I	Function II	Function III
0.99200	0.80700	0.61100
0.00100	1.29900	0.38200
0.02100	0.16200	1.14800

Table 2

Cosines Among Factor Axes Resulting From Procrustean Rotation Invariance Procedure, With Test r's

	Function I	Function II	Function III
Function I	**0.9978**	0.0006	0.0661
Function II	0.0145	**0.9776**	0.2098
Function III	0.0645	0.2103	**0.9755**

Note. TEST r's were 0.9971, 0.9979, and 0.9979.